

*AN EXPERIMENTAL INVESTIGATION OF THE  
VISCOELASTIC PROPERTIES OF SMALL  
VOLUMES OF MATERIAL*

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# MOTIVATION

## ❖ *What we're after:*

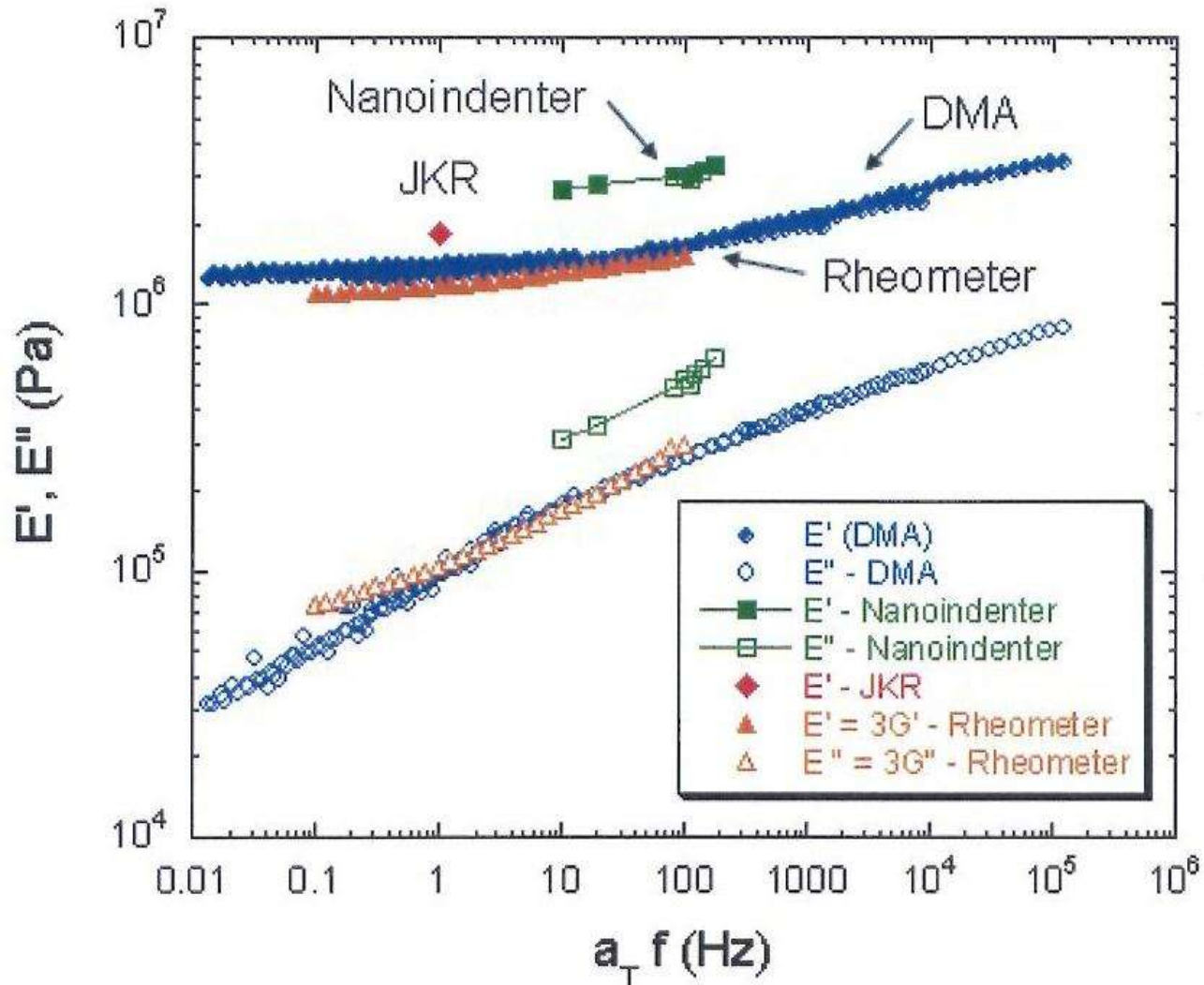
- Constitutive behavior of small volumes of viscoelastic solids subjected time varying excitation over as wide a range of time or frequency as possible.

## ❖ *Extend the applicability of nanoindentation to time-dependent behavior*

- *Flat punch indentation, complex test geometry*
- *DMA, triple clamp fixture, complex geometry*
- *Simple test geometry, uniaxial compression*

- Material's response in the frequency domain - short time
- Material's response in the time domain - long time
- Bringing them together
- Do it all with flat punch indentation
- Material selection: [Highly plasticized PVC](#)

# NANOINDENTATION & DMA COMPARISON



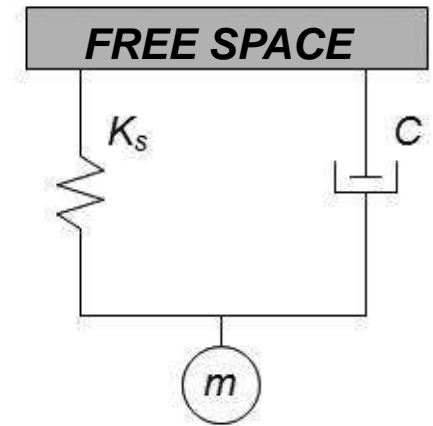
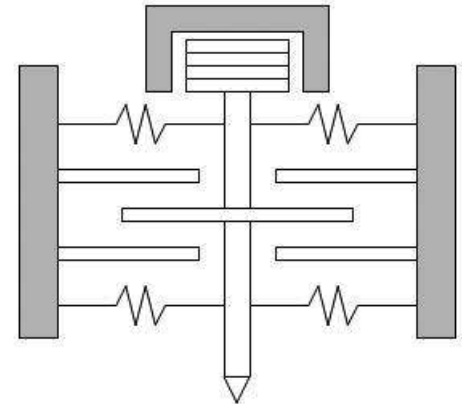
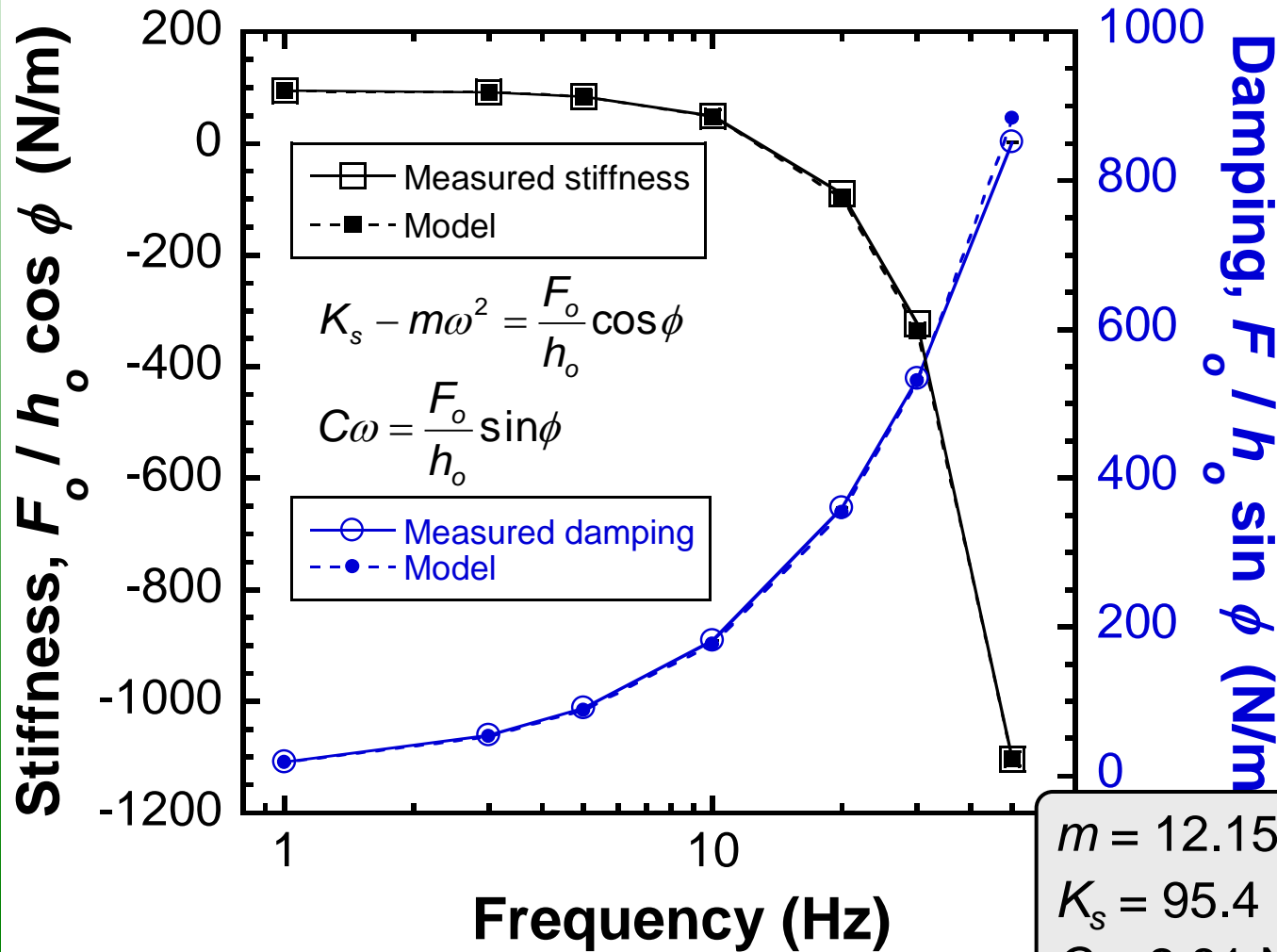
C. C. White et al., Mater. Res. Soc. Symp. Proc. **841** (2005)



# MODELING THE INSTRUMENTATION

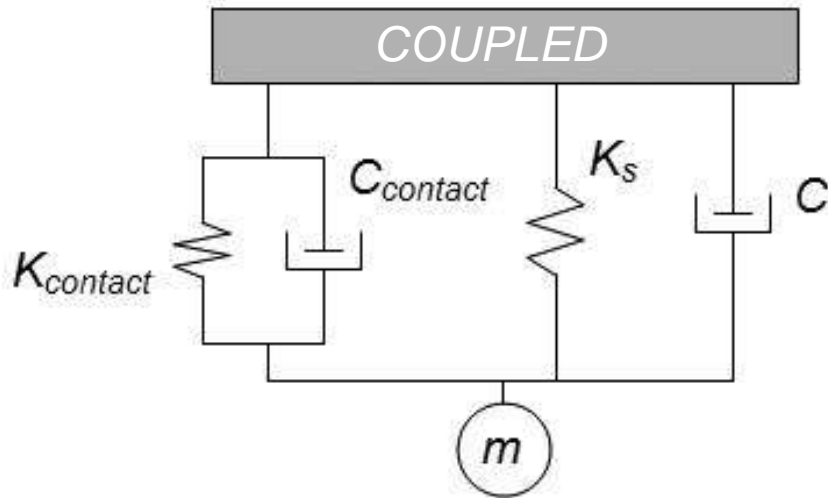
- Measure time-dependent material properties
- Then we need to understand the time-dependent properties of the measurement tool

# Measured stiffness and damping in free space, position = 18.8 $\mu\text{m}$



$m = 12.15 \text{ g}$   
 $K_s = 95.4 \text{ N/m}$   
 $C = 2.81 \text{ Ns/m}$

# ADD THE CONTACT



$$S = \frac{F_o}{h_o} \cos \phi + m\omega^2$$

$$C = \frac{F_o}{h_o} \frac{\sin \phi}{\omega}$$

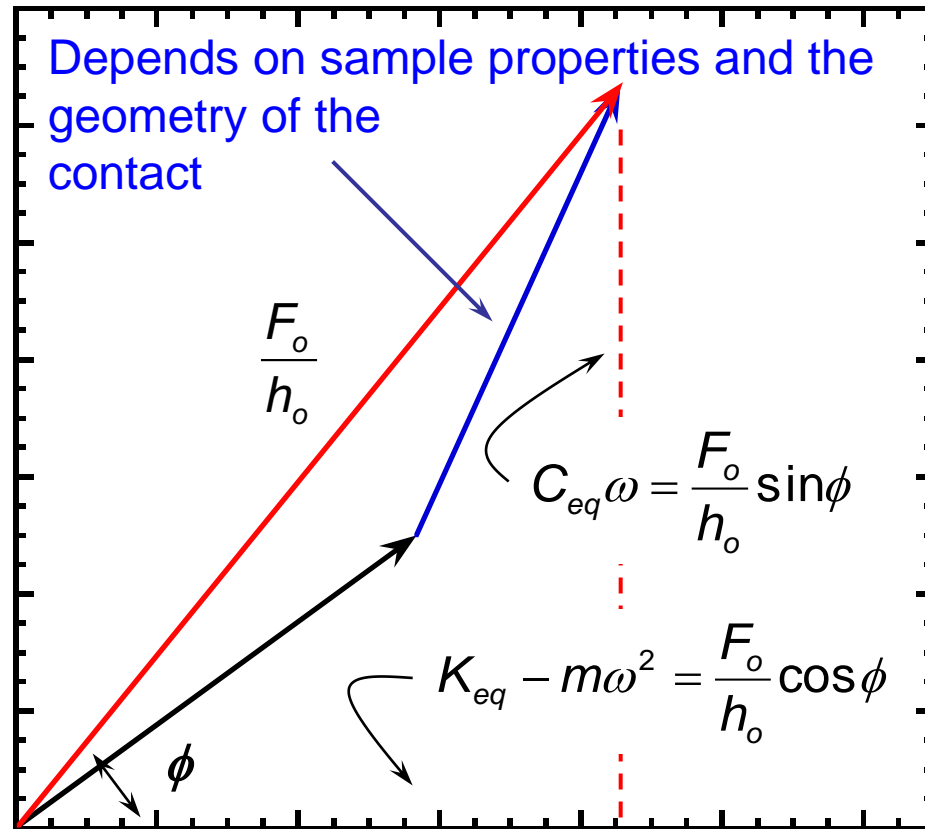
**COUPLED RESPONSE = SAMPLE + INSTRUMENT**

$$K_{contact} = \left[ \frac{F_o}{h_o} \cos \phi + m\omega^2 \right]_{\text{coupled}} - \left[ \frac{F_o}{h_o} \cos \phi + m\omega^2 \right]_{\text{inst. (free space)}}$$

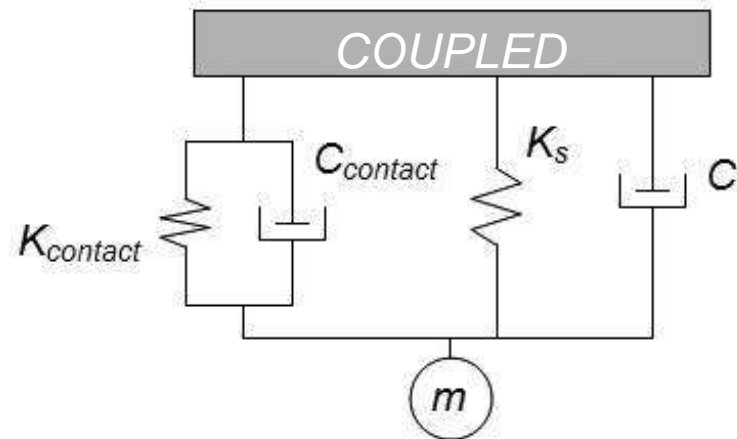
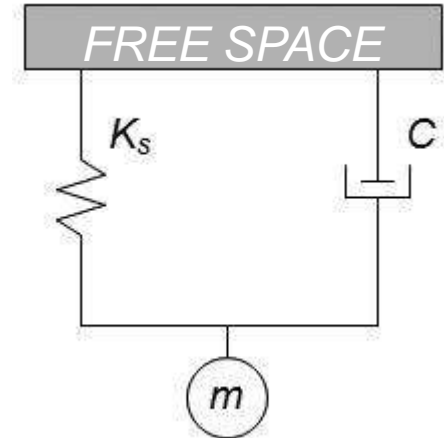
# PHASOR DIAGRAM: PHYSICAL INSIGHT

## Damped, forced oscillator

Imaginary axis (damping,  $C_{eq}\omega$ , N/m)



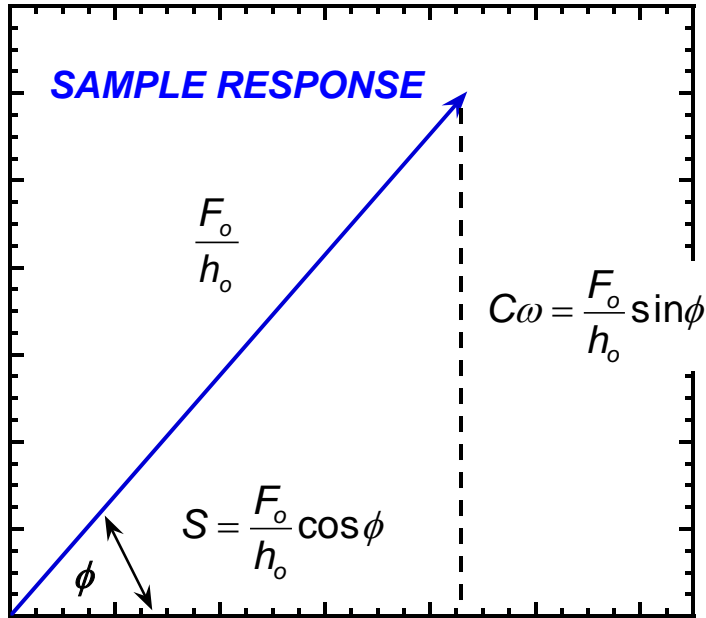
Real axis (stiffness,  $K_{eq} - m\omega^2$ , N/m)



# FROM SAND $C\omega \rightarrow E'$ AND $E''$

Imaginary axis (damping,  $C\omega$ , N/m)

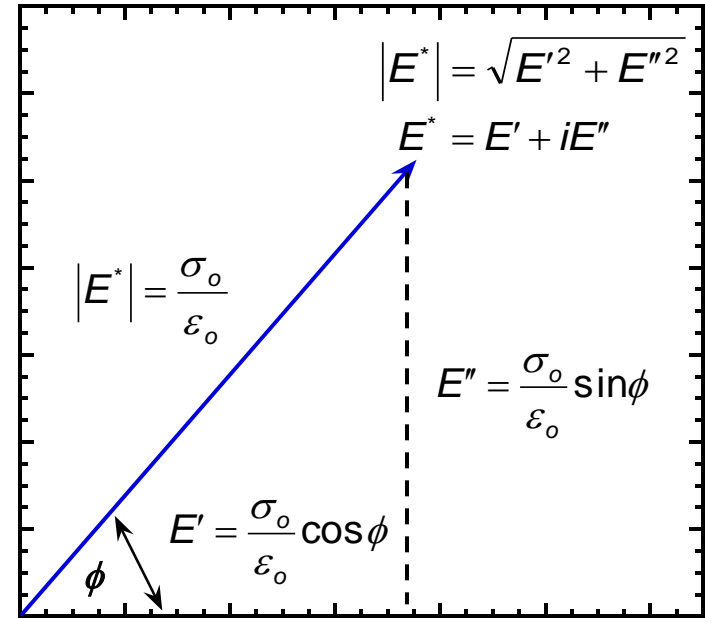
Phasor diagram of experimental measurements



Real axis (stiffness,  $S$ , N/m)

Imaginary axis (viscous stress, Pa)

Phasor diagram of a linear viscoelastic solid



Real axis (elastic stress, Pa)

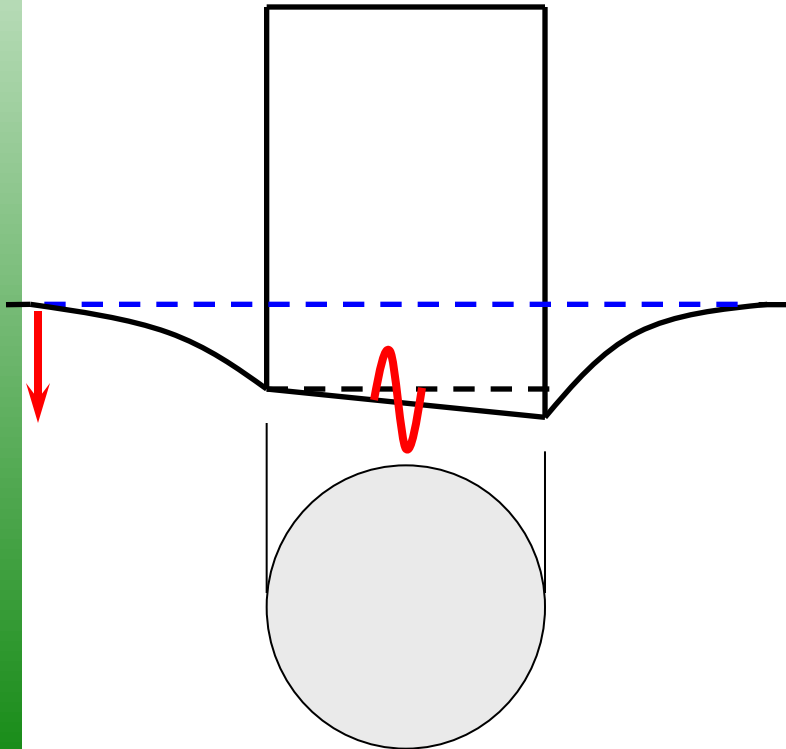
**The fundamental equation of nanoindentation:**

$$E' = S \frac{\sqrt{\pi}}{2} \frac{1}{\beta} \frac{1}{\sqrt{A}}$$

$$E'' = C\omega \frac{\sqrt{\pi}}{2} \frac{1}{\beta} \frac{1}{\sqrt{A}}$$

# GEOMETRY OF THE CONTACT

*Circular flat punch:*



## ***Advantages:***

- Known contact area
- Area not affected by creep or thermal drift

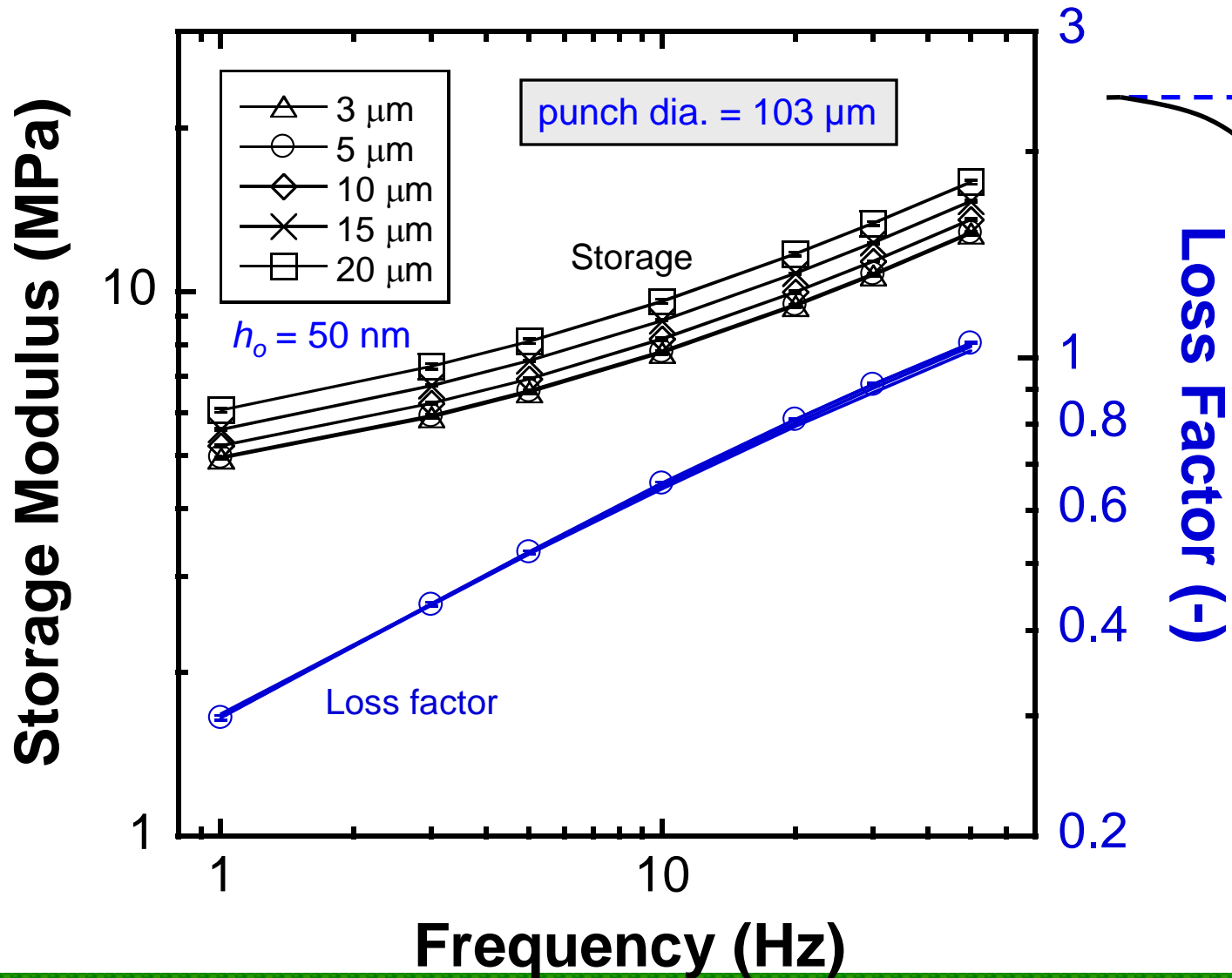
## ***Disadvantages:***

- Full contact
- Stress concentration

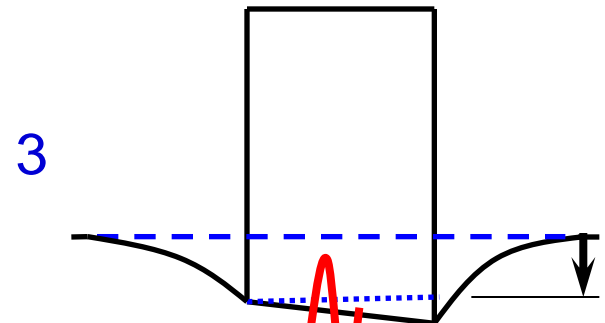
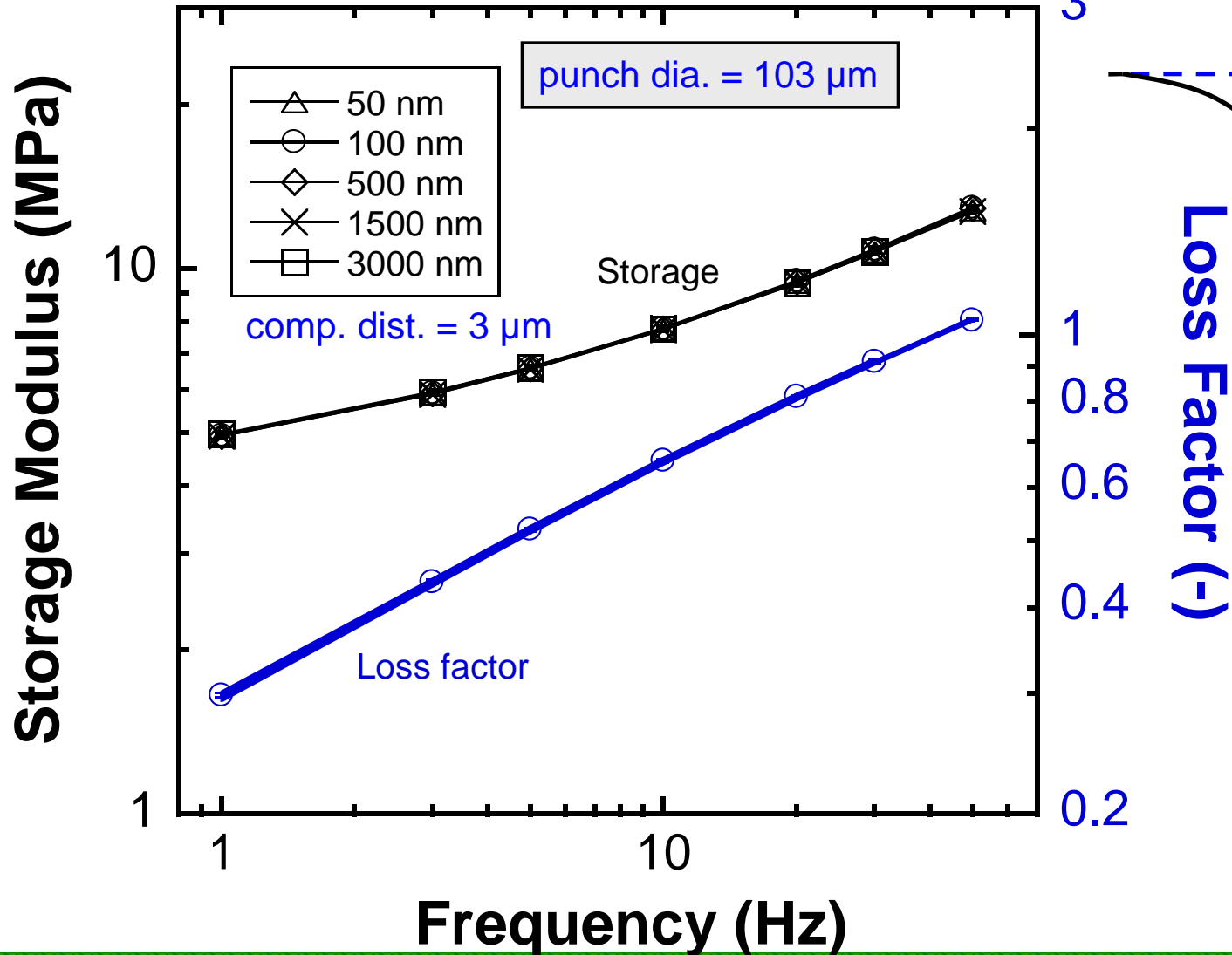
## ***Any tip geometry, consider:***

- Steady-state harmonic motion
- Linear viscoelasticity
  - Compression distance
  - Oscillation amplitude
- Loading rate

# Pre-Compression Dependence



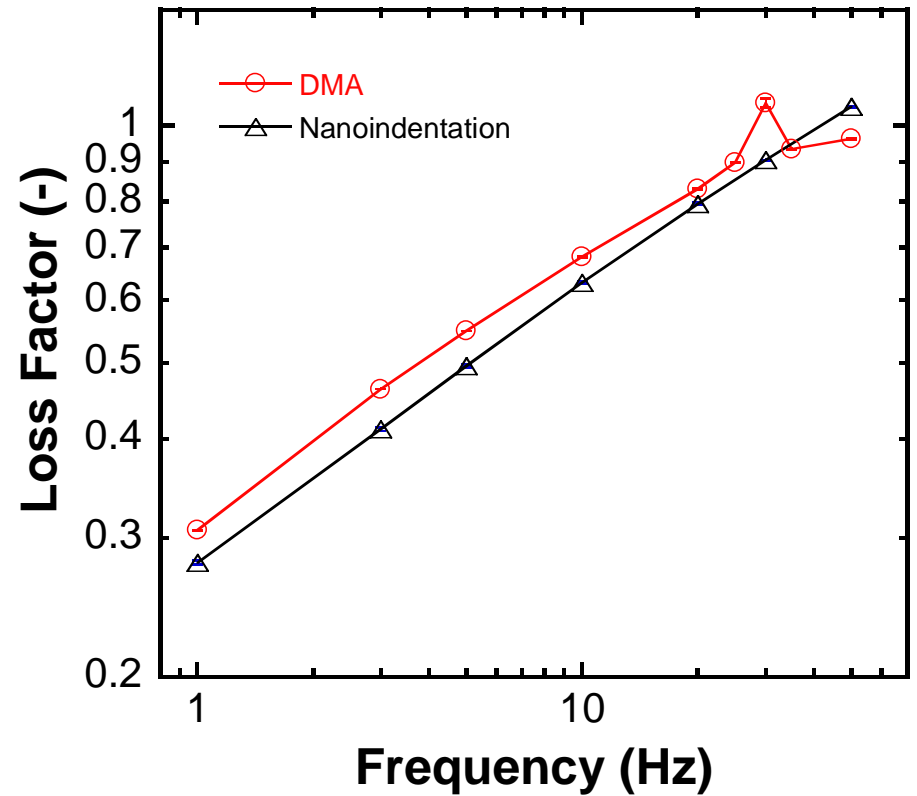
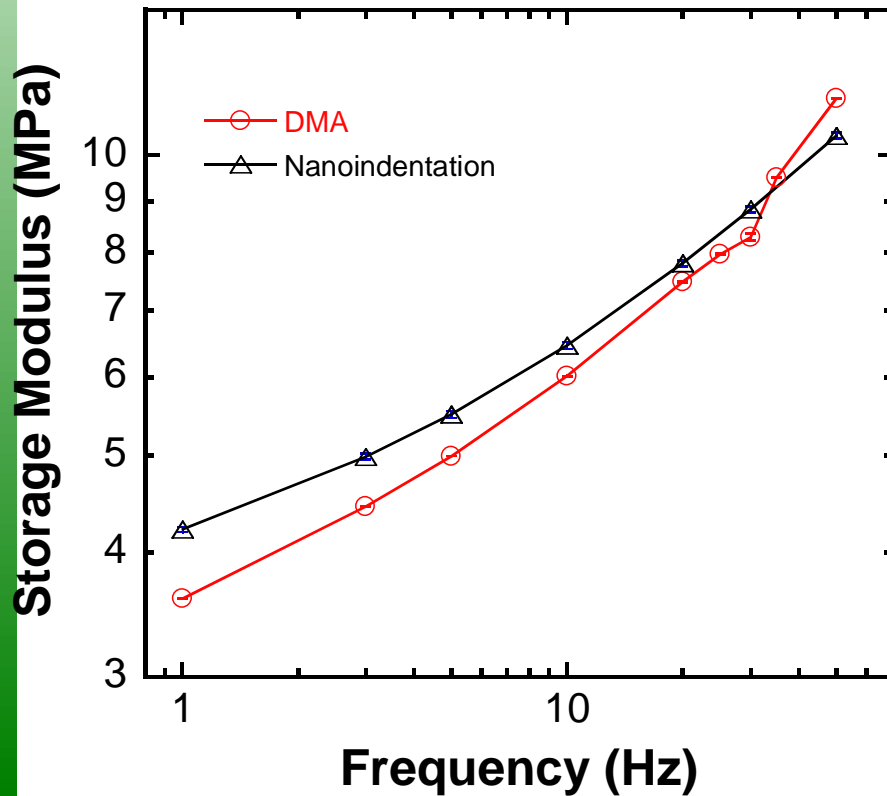
# Amplitude Independence



Loss Factor (-)

# DMA VS. NANOINDENTATION

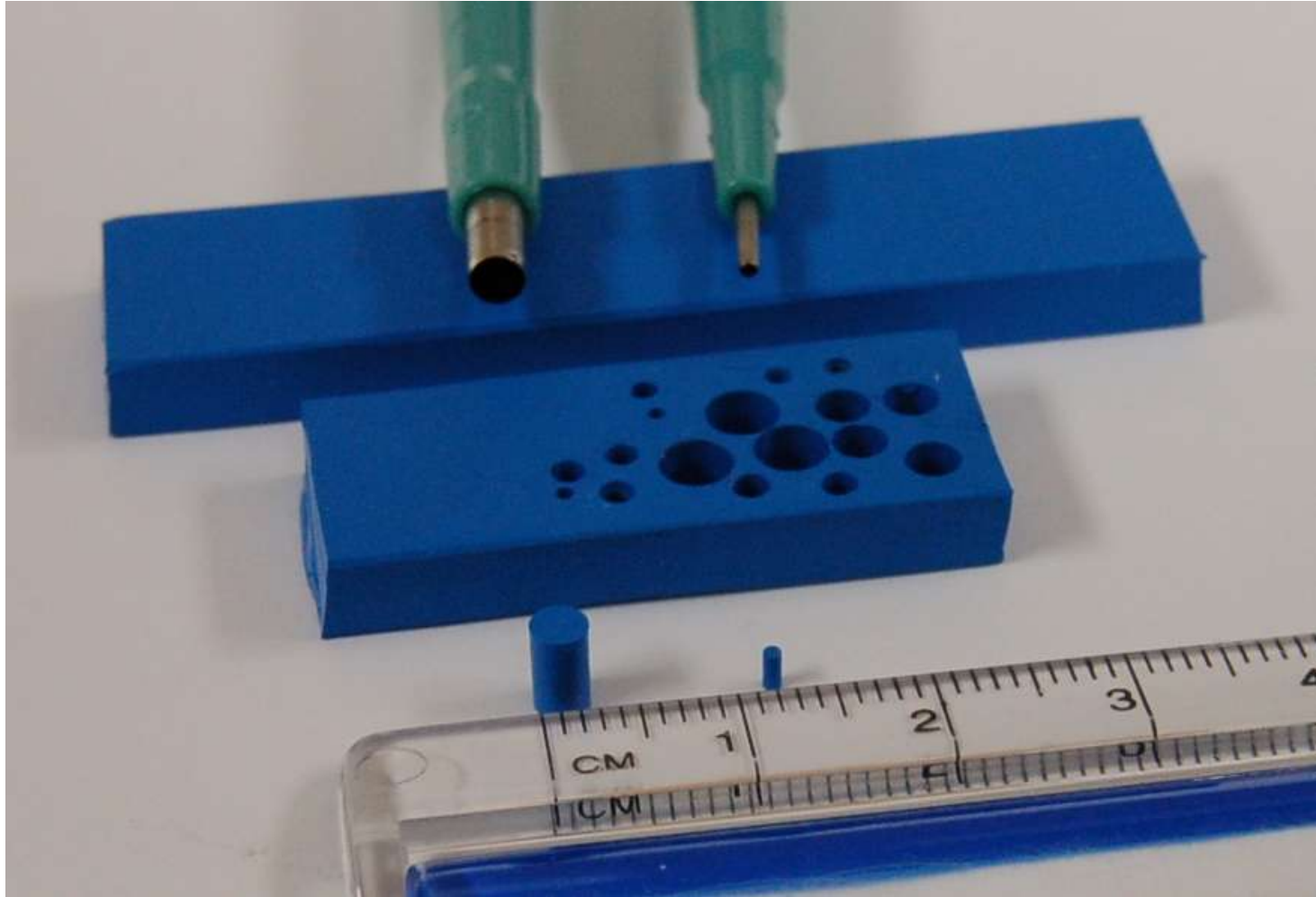
Highly plasticized polyvinylchloride,  
the complex modulus at 22 °C



## NEXT STEPS

- Do a better job comparing flat punch experiments to tests with simpler geometry.
- Study creep compliance with a flat punch.
- Combine frequency specific results with creep compliance.

# COMPRESSION SAMPLES



# COMPRESSION & INDENTATION

$$E' = \frac{F_o}{h_o} \cos \phi (\text{geometry factor})$$

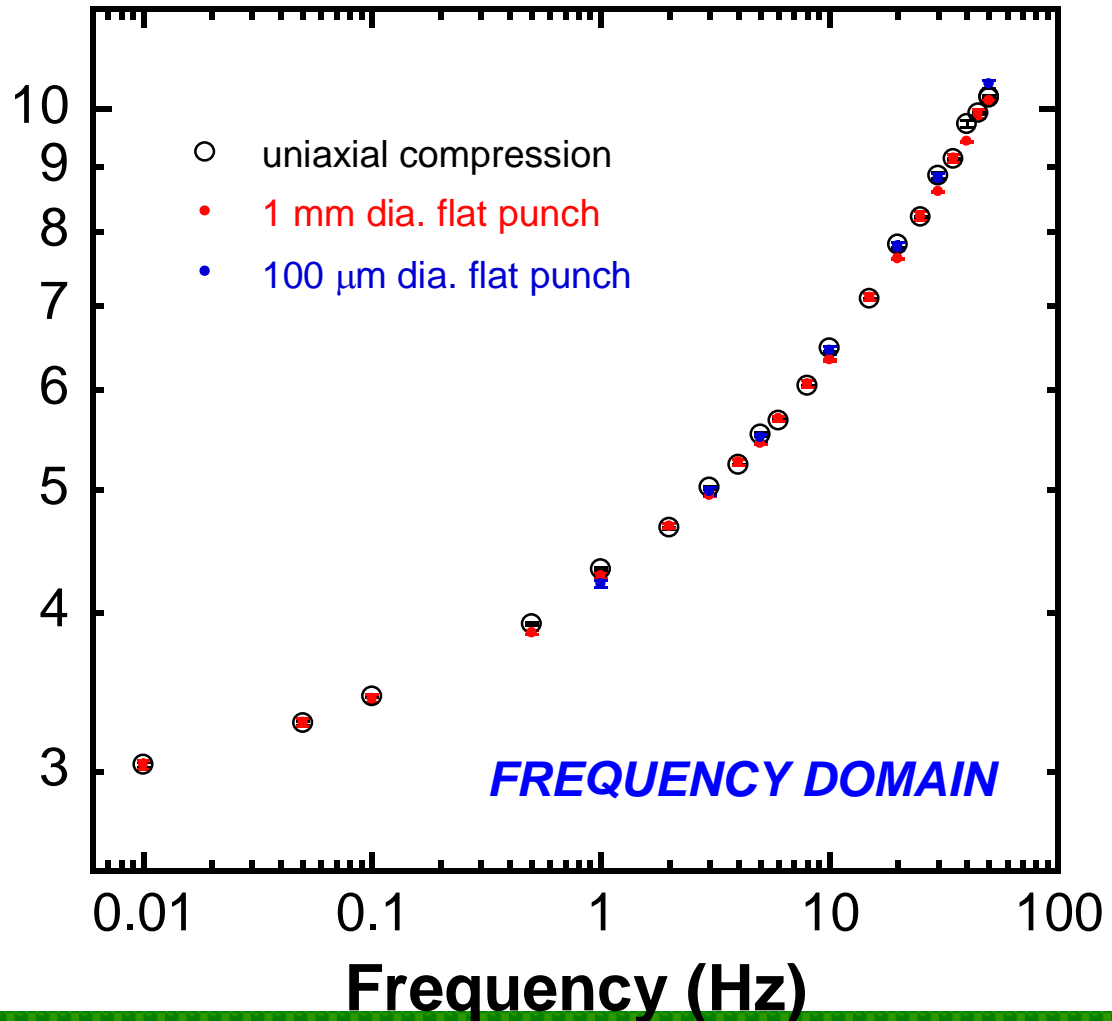
$$E'' = \frac{F_o}{h_o} \sin \phi (\text{geometry factor})$$

Geometry factors:

Compression:  $\frac{L}{A}$

Indentation:  $\frac{\sqrt{\pi}}{2} \frac{1}{\beta} \frac{1}{\sqrt{A}} (1-\nu^2)$

**$E'$  (MPa)**



# COMPRESSION & INDENTATION

$$E' = \frac{F_o}{h_o} \cos \phi (\text{geometry factor})$$

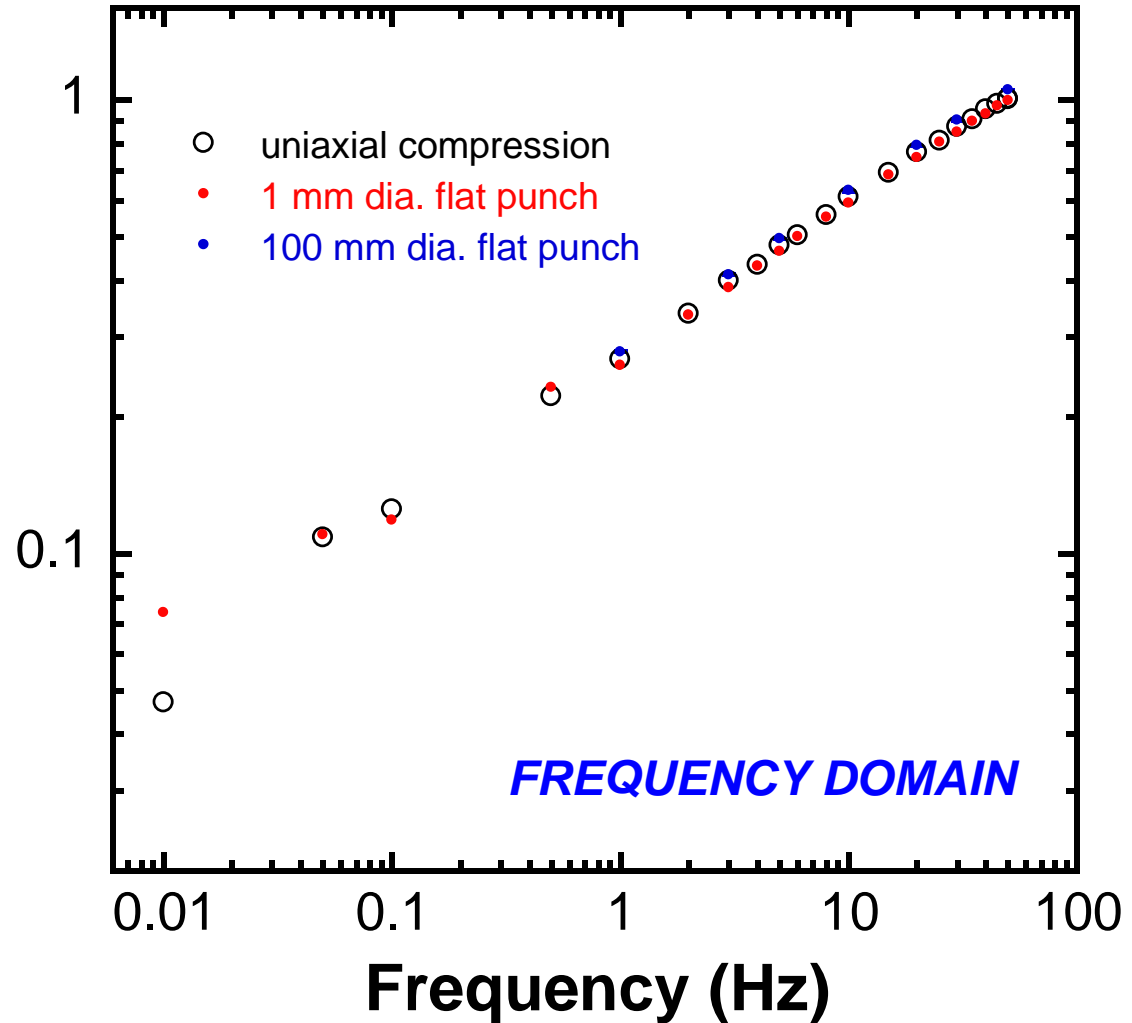
$$E'' = \frac{F_o}{h_o} \sin \phi (\text{geometry factor})$$

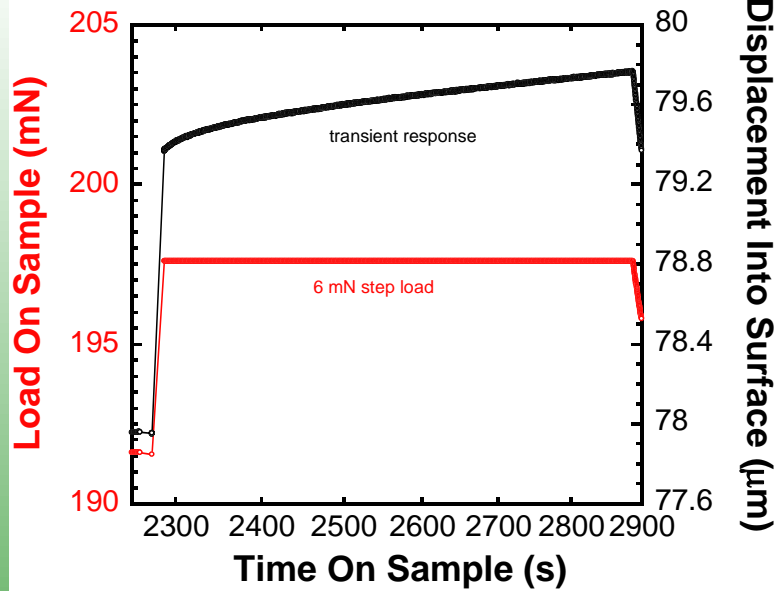
Geometry factors:

$$\text{Compression: } \frac{L}{A}$$

$$\text{Indentation: } \frac{\sqrt{\pi}}{2} \frac{1}{\beta} \frac{1}{\sqrt{A}} (1 - \nu^2)$$

Loss Factor (-)





$$D(t) = \frac{\varepsilon(t)}{\sigma_o}$$

Compression:

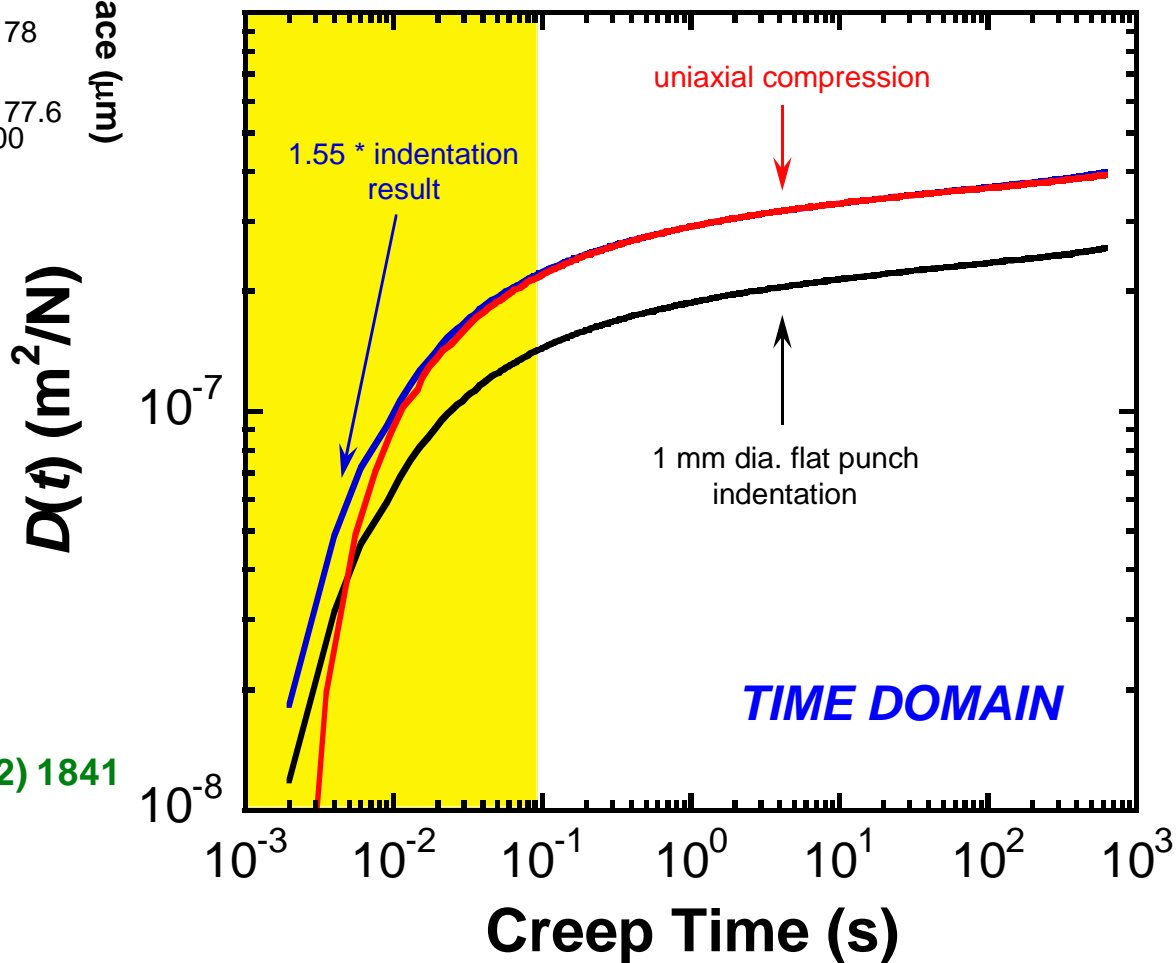
$$\sigma = \frac{P}{A} \quad \varepsilon = \frac{\Delta L}{L}$$

$$D(t) = \frac{A \Delta L}{L P}$$

Indentation:

$$\sigma \propto H = \frac{P}{A} \quad \varepsilon \propto \frac{h}{D}$$

$$D(t) = C \frac{h}{H D}$$



Sakia, M., Phil. Mag. A 82 10 (2002) 1841

# TRANSFORMING FROM FREQUENCY TO TIME

**4 term Prony series:**

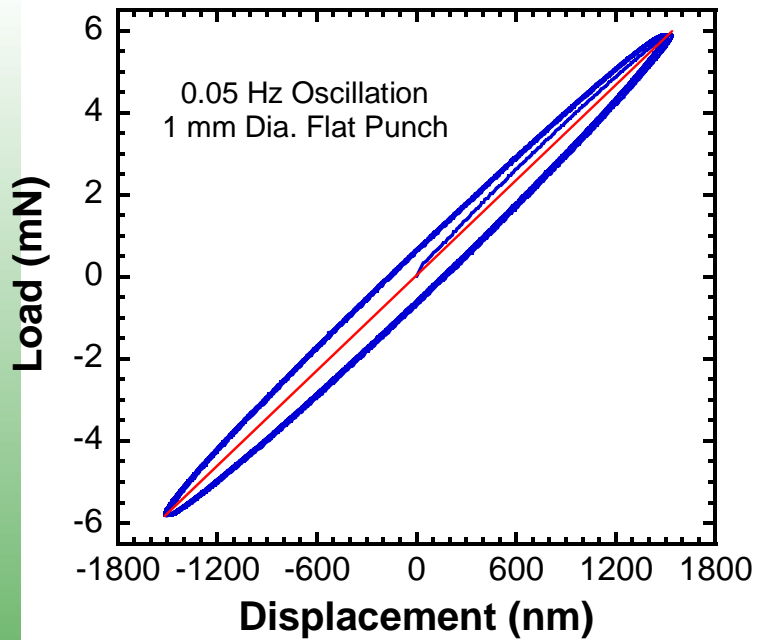
**FREQUENCY DOMAIN:**

$$J' = J_0 + \sum_{i=1}^4 \frac{J_i}{1 + \tau_i^2 \omega^2}$$

$$J' = \frac{E'}{E'^2 + E''^2}$$

**TIME DOMAIN:**

$$D(t) = j_0 + \sum_{i=1}^4 J_i (1 - e^{-\frac{t}{\tau_i}})$$



$$E' = \frac{F_o}{h_o} \cos \phi (\text{geometry factor})$$

$$E'' = \frac{F_o}{h_o} \sin \phi (\text{geometry factor})$$

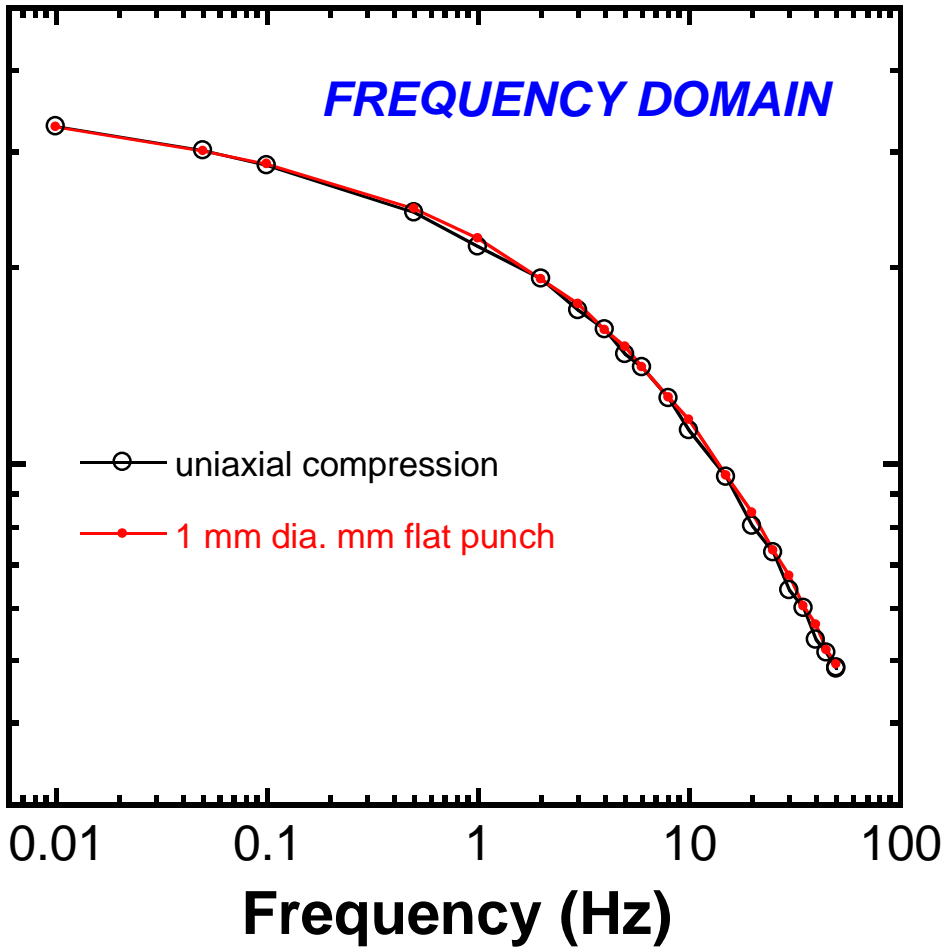
$$J' = \frac{E'}{E'^2 + E''^2}$$

Geometry factors:

Compression:  $\frac{L}{A}$

Indentation:  $\frac{\sqrt{\pi}}{2} \frac{1}{\beta} \frac{1}{\sqrt{A}} (1 - \nu^2)$

$J'$  ( $\text{m}^2/\text{N}$ )

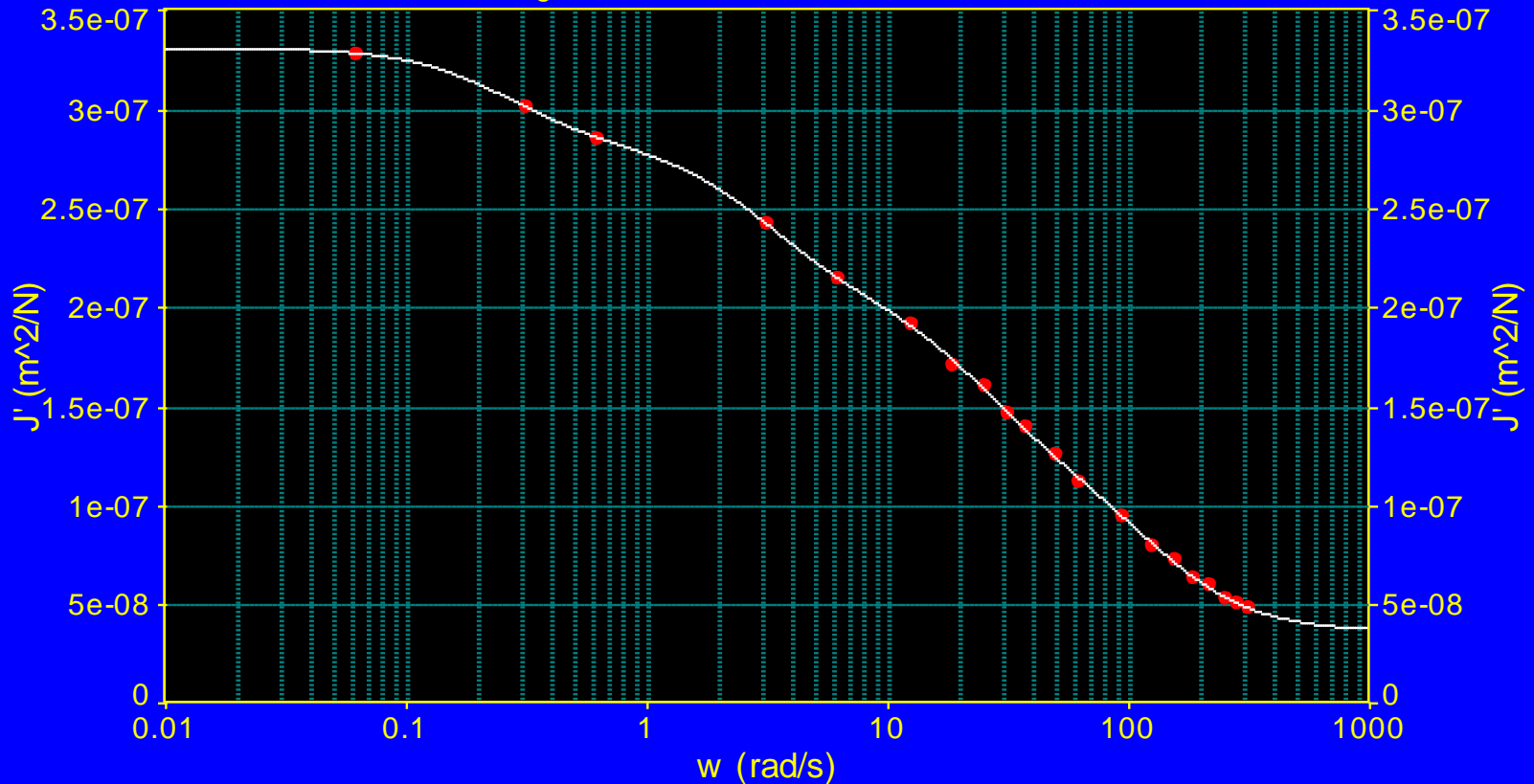


# J' (FREQ. DOMAIN) FIT TO PRONY SERIES

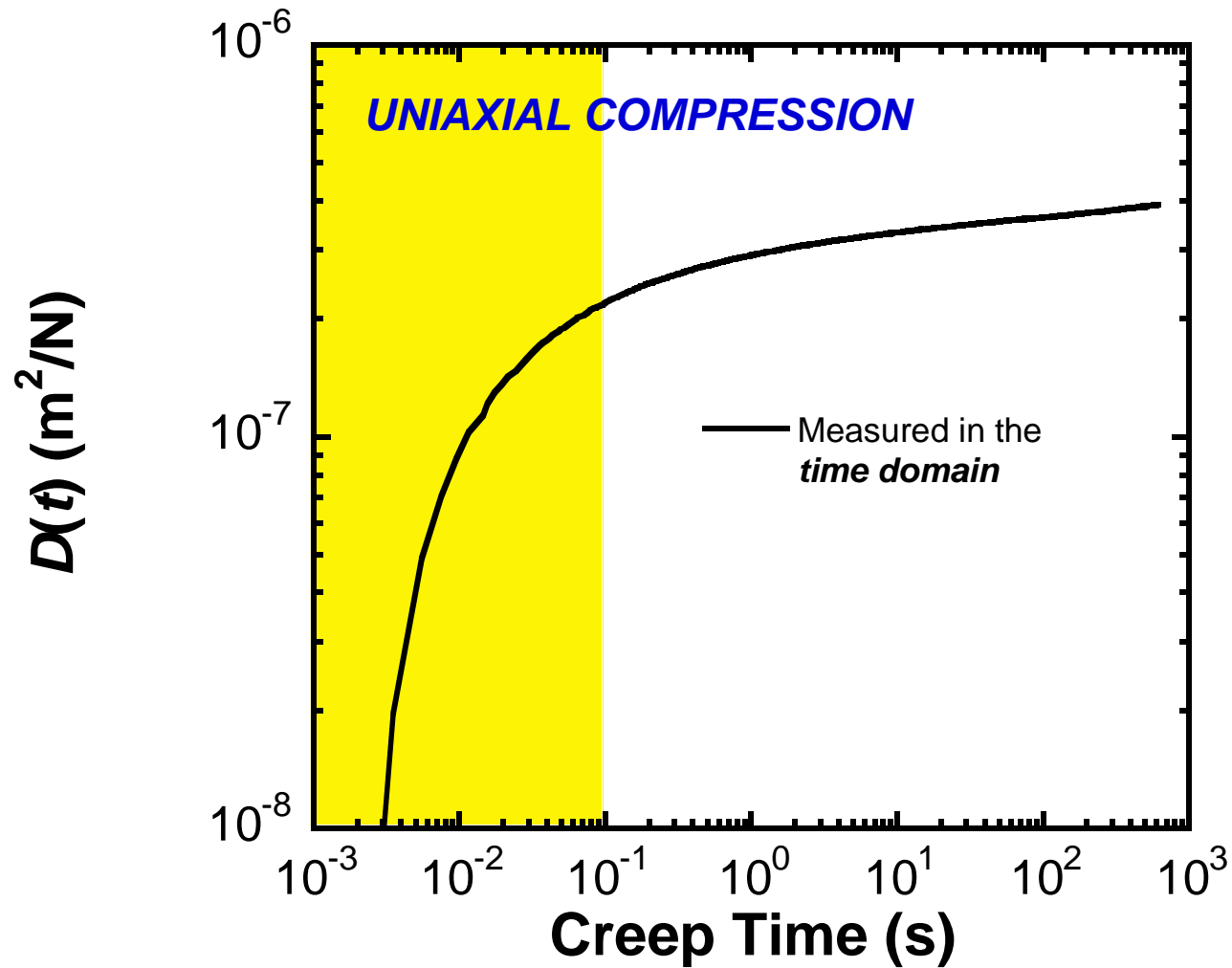
## UNIAXIAL COMPRESSION

Eqn 8001 Prony 4(a,b,c,d,e,f,g,h,i)

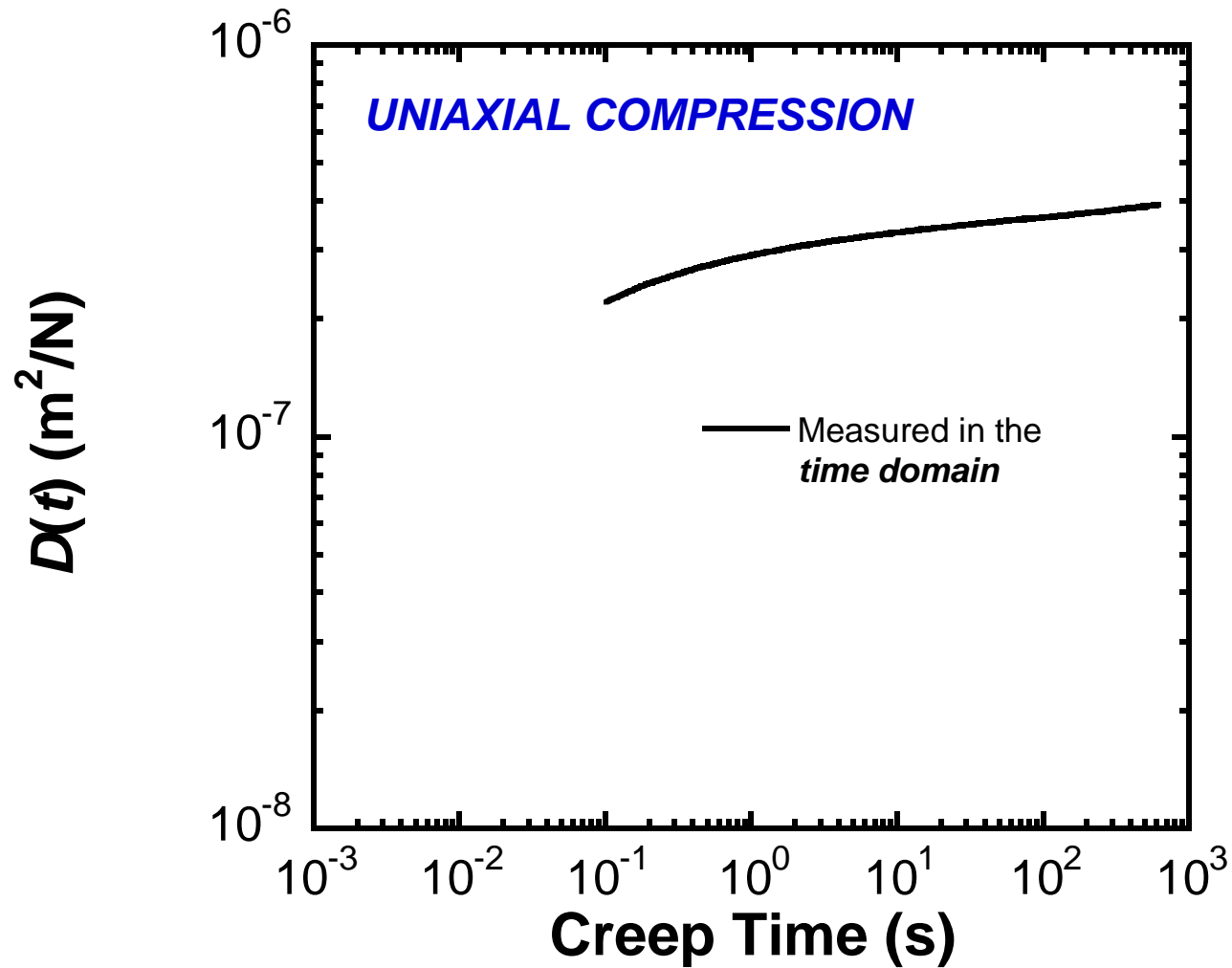
$r^2=0.99984539$  DF Adj  $r^2=0.99970624$  FitStdErr= $1.4493252e-09$  Fstat= $8891.8589$   
a= $3.6679128e-08$  b= $8.0587677e-08$  c= $-0.0077065727$  d= $5.0195154e-08$  e= $3.6800044$   
f= $8.6739055e-08$  g= $-0.039185236$  h= $7.724322e-08$  i= $-0.30676693$



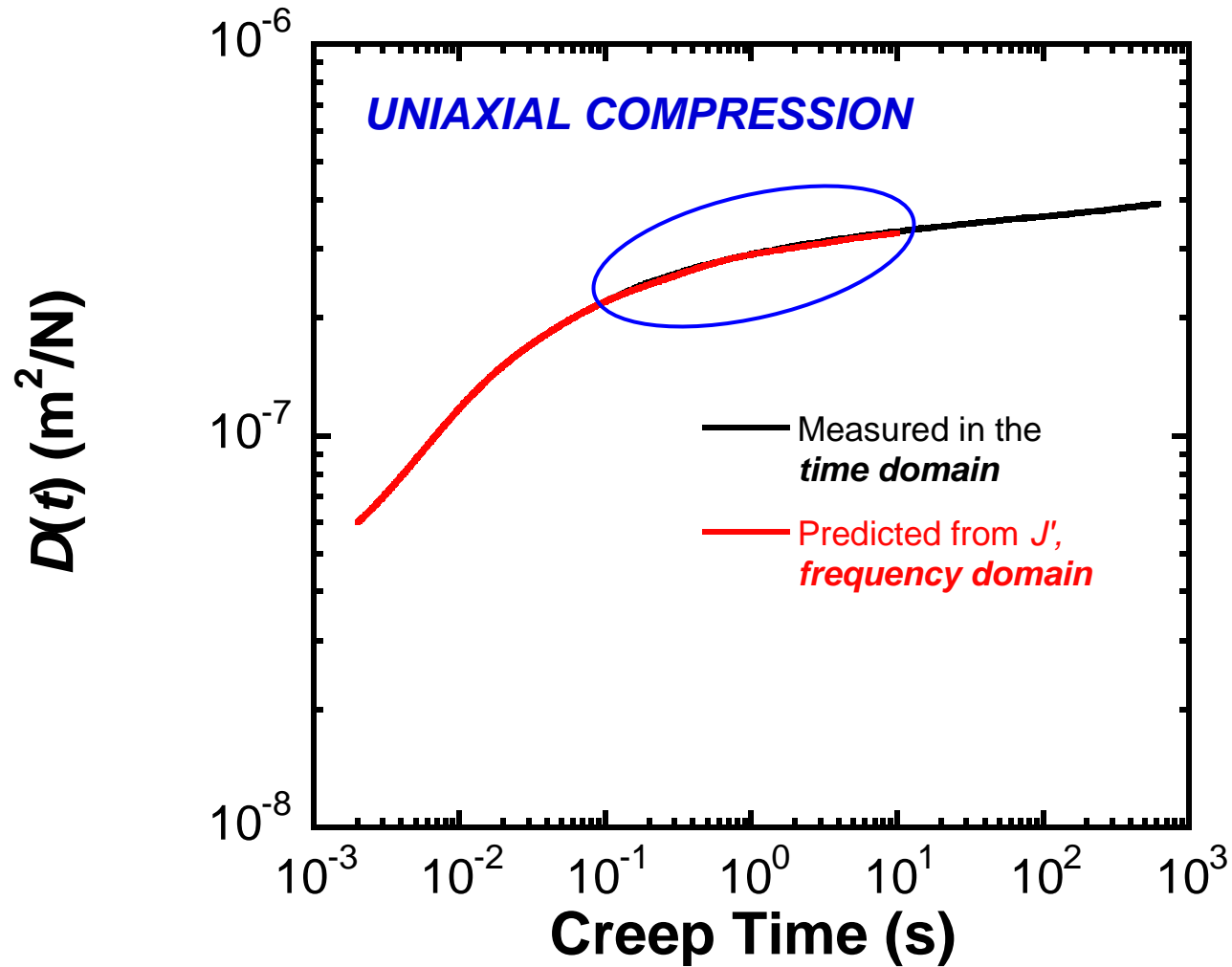
# VERIFICATION OF THE PATH FROM $J'$ TO $D(t)$



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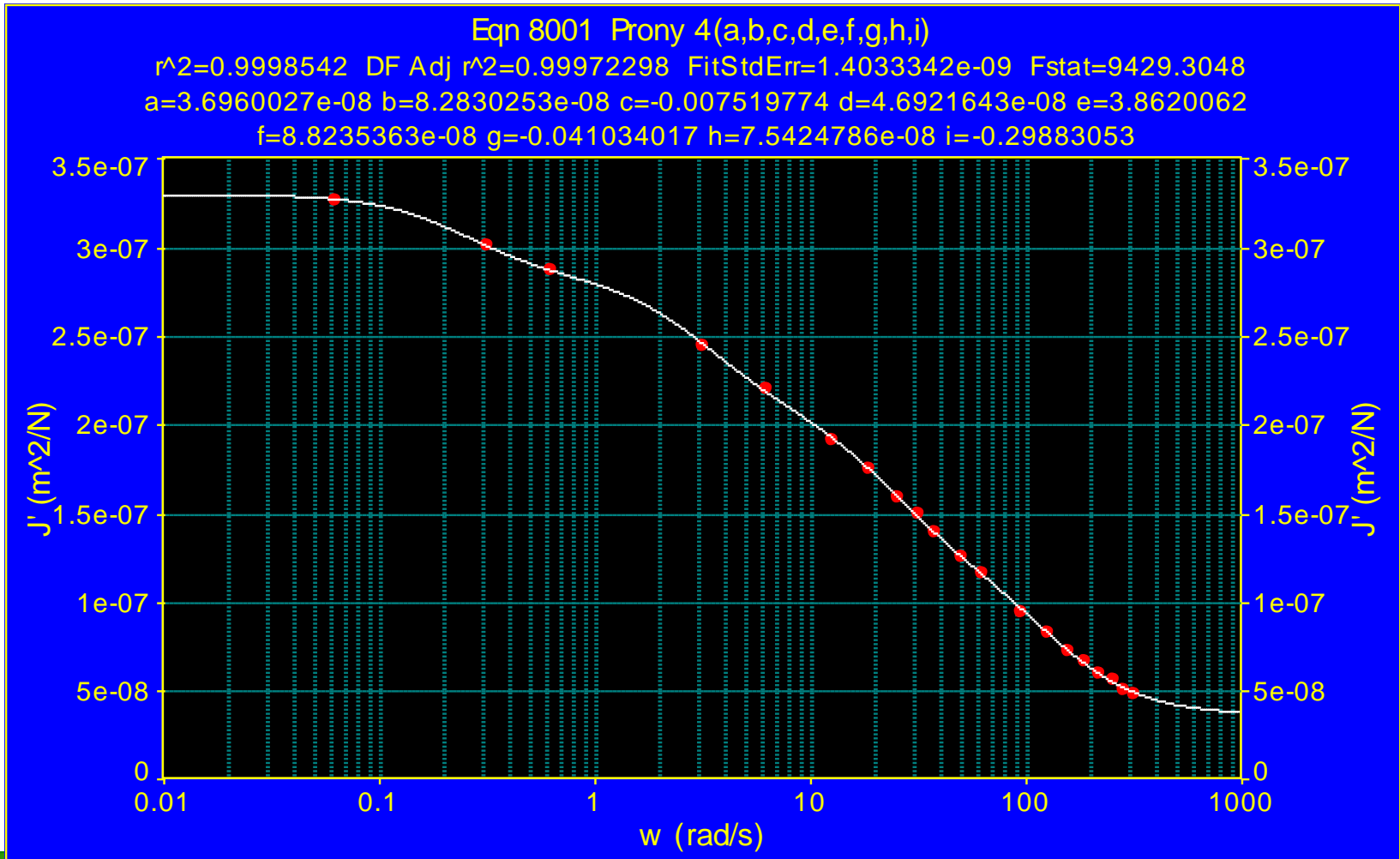


# VERIFICATION OF THE PATH FROM $J'$ TO $D(t)$

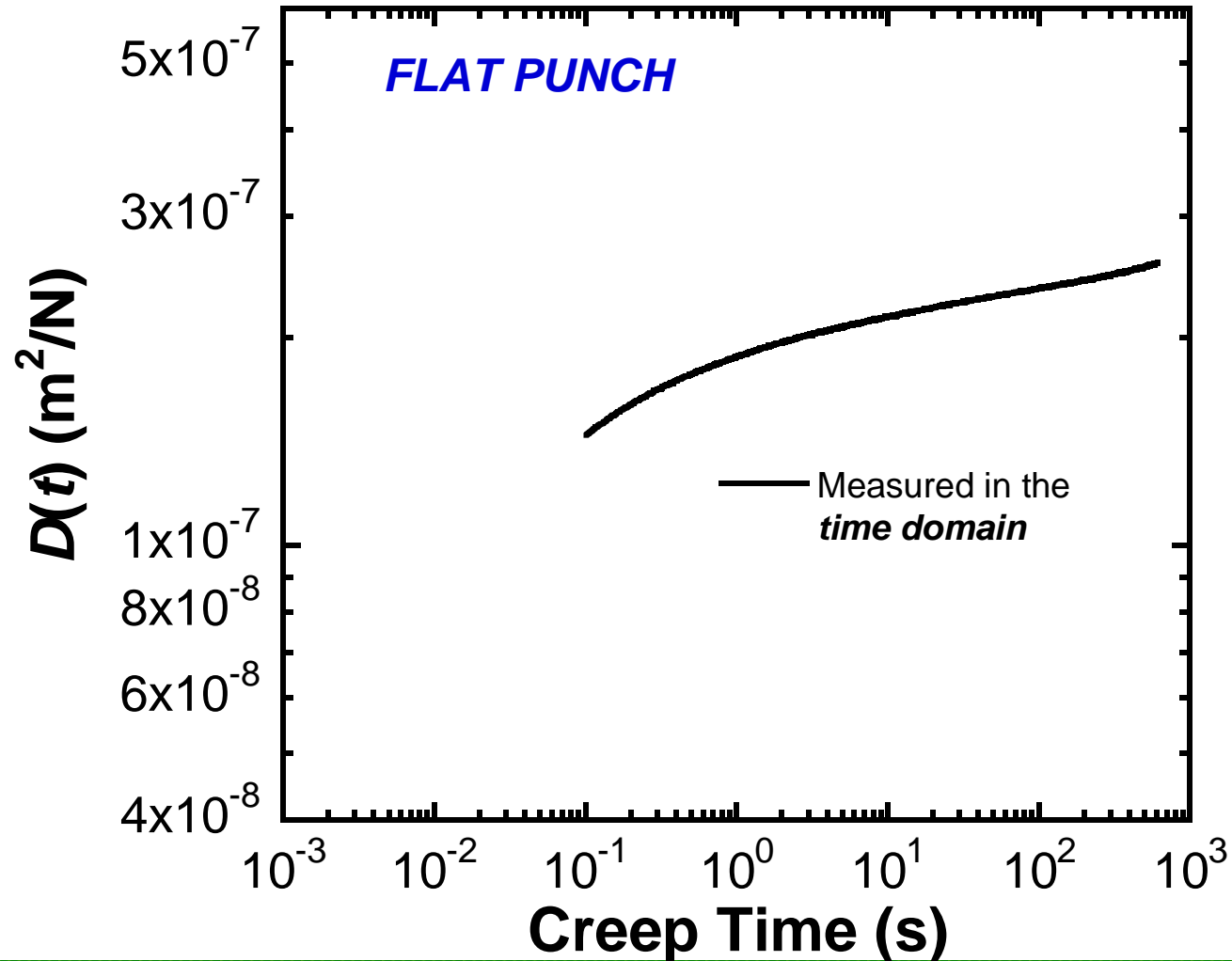


# J' (FREQ. DOMAIN) FIT TO PRONY SERIES

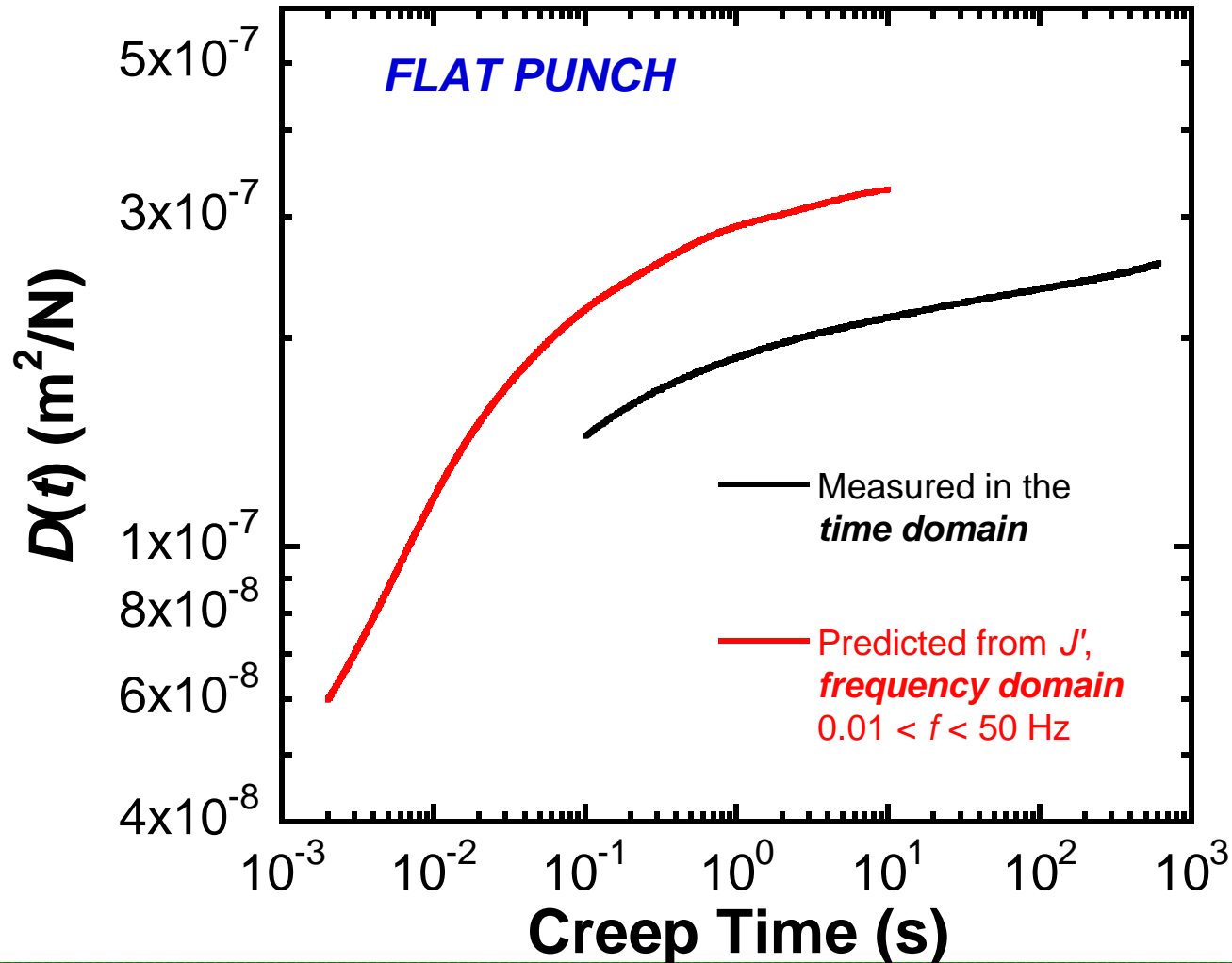
## *FLAT PUNCH INDENTATION*



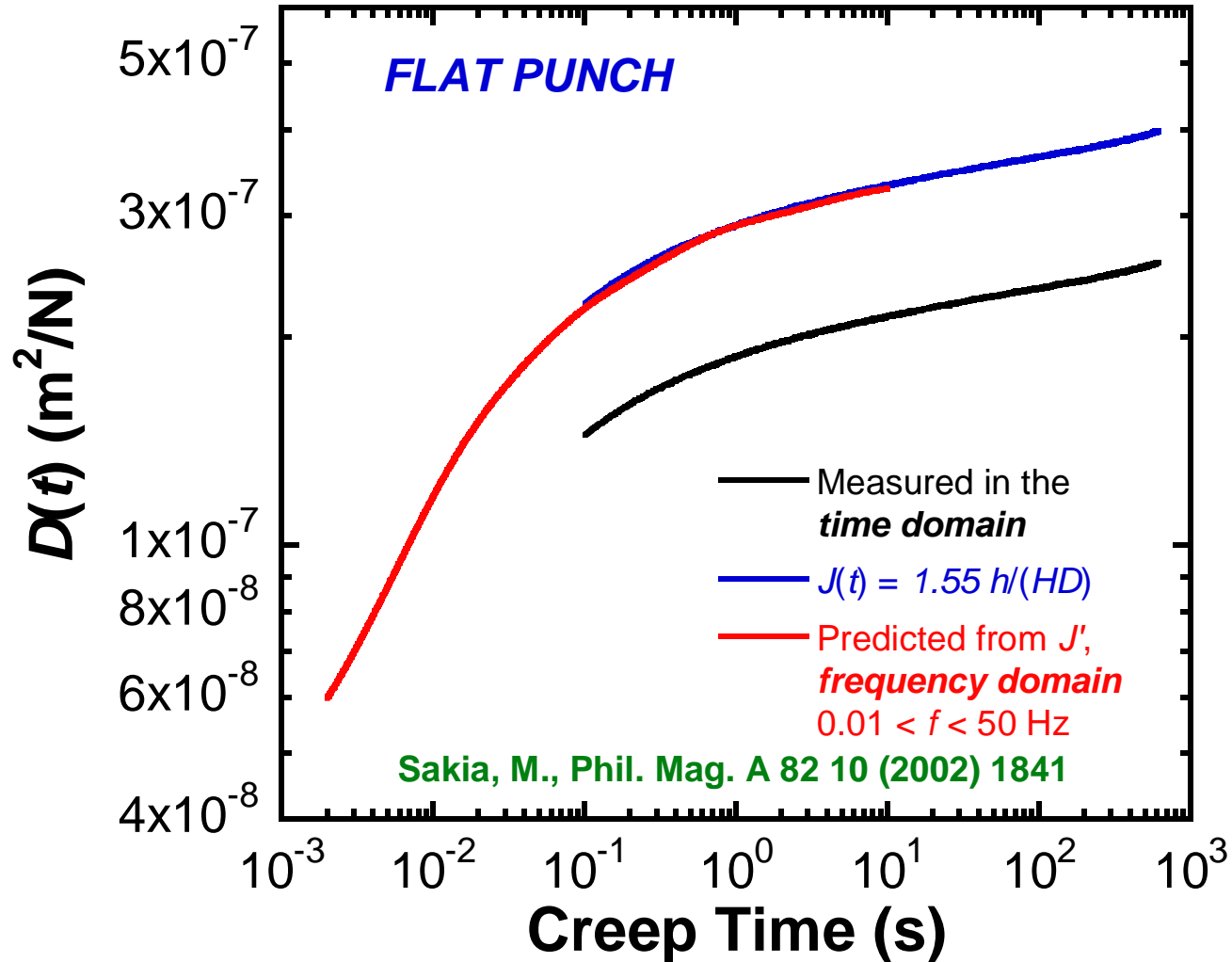
# MAXIMIZING THE TIME AND FREQUENCY RANGE



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# CONCLUSIONS

- ❖ Dynamic nanoindentation of viscoelastic solids requires *robust dynamic characterization of the measurement tool itself*, a *known contact geometry*, *steady-state harmonic motion*, and *linear viscoelasticity*.
- ❖ In the frequency domain, Sneddon's stiffness equation works remarkably well.
- ❖ The Prony series model provides a valid path to transition between the frequency and time domains.
- ❖ It is possible to combine frequency and time domain data from a flat punch indentation experiment and therefore characterize the sample's behavior over the widest possible range of time and frequency.
- ❖ Sakia's calculation of the geometric factor relating flat punch time domain results to uniaxial results is confirmed.